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LOCAL FORECAST OF THE HEAT-MOISTURE STATE OF THE SOIL SURFACE AS A PROBLEM OF ENERGY AND MASS TRANSFER IN THE SYSTEM OF SOIL–ATMOSPHERE

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Consideration is given to the dynamic problem of forecasting a frost on the soil surface, namely, of establishing the fact and the moment of the water–ice and water vapor–ice phase transitions at the soil–atmosphere interface, in calculating the evolution of the heat-moisture state of the viscous-buffer layer of air and the near-surface layer of soil that are adjacent to this interface. The results obtained can be used for prediction of ice formation on runways of airfields and on roads.

Reliable prediction of such critical events as soil frosts and glazed frost on roads and on runways of airfields is an important problem in providing stable functioning of agricultural production and safe service of motor-vehicle and aircraft transport. The available methods, algorithms, and mathematical models of forecast of these events are empirical and represent regression relations, graphs, or charts obtained, as a rule, by statistical processing of long-term meteorological observations [1–5]. They are correlated with the place to which the forecast pertains; therefore their dissemination is diffucult, their accuracy is low, and the longer-term the prediction, the lower the accuracy. At the same time, the need for universal high-accuracy algorithms of forecast of the considered events with a probability of their occurrence no lower than 0.85 is dictated by practical needs and, in particular, by the safety and regularity of passenger and freight motor- and air-transportation [3].

In the present work, prediction of ice formation on the surface of soil, a road, or a runway is formulated as the problem of the evolution of the thermodynamic state of the viscous-buffer layer of air and the near-surface layer of soil (a road, a runway) adjacent to the hyperplane of the soil–atmosphere interface and of establishing the fact and the moment of water–ice or vapor–ice phase transitions on the indicated hyperplane in the course of this evolution. In this formulation, forecast of the events under consideration represents an initial-boundary-value problem with free conditions at the internal boundary, which is incorrect [6] for dynamic models of interaction in the system of soil–atmosphere.

Interaction between the Near-Ground Atmospheric Layer and a Rough Soil Surface. We will define the viscous-buffer layer of the atmosphere in the neighborhood of the forecast point as an immobile, on the average (in the statistical sense [7]), air sublayer of thickness z_*–z_{**} (z_* and z_{**} are the levels of the dynamic local landscape and surface-soil roughnesses in the neighborhood of the forecast point) adjacent to the soil surface, i.e., a layer in which $\overline{u(z)} = 0$, $z \in [z_{**}, z_*]$, $\overline{u}$ is the average horizontal component of the vector of the wind velocity. The level z_* and the other dynamic characteristics of the turbulent shear flow of air above the level z_* (the scales of length L, velocity $\hat{u}_*$, temperature $\hat{T}_*$, and moisture $\hat{q}^*$ and the quantities $T(z_*)$ and $q(z_*)$) are found by solution of the problem of the interaction of the turbulent boundary layer of the atmosphere with the local landscape, agrolandscape, or homogeneous vegetation as the layer of permeable

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roughness with a known volume thickening (see, for instance, [8]). Evaluation of the dimensions of the neighborhood of the forecast point is beyond the scope of the present article. Below for simplicity's sake we assume that the neighborhood of the forecast point is free of vegetation and has a plain landscape with a small difference of heights.

The interaction of the near-ground atmospheric layer with a rough soil surface in the neighborhood of the forecast point can be described by universal functions. Choosing as such the "perturbed logarithm" [9–11], we have

$$u(z) = \hat{u}_* \kappa^{-1} \left[\ln \frac{z}{z_*} + \varphi_1 \right], \; T(z) = T(z_*) + \hat{T}_* c_1^{-1} \left[\ln \frac{z}{z_*} + \varphi_2 \right], \; q(z) = q(z_*) + \hat{q}_* c_2^{-1} \left[\ln \frac{z}{z_*} + \varphi_3 \right]; \tag{1}$$

$$\varphi_i = \int \xi^{-1} (1 - \Phi_i(\xi))\, d\xi, \quad i = 1, 2, 3; \tag{2}$$

$$\Phi_1 = \begin{cases} (1 - 15)^{-1/4}, & \xi < 0, \\ 1 + 4.7\xi, & 0 < \xi < 1, \\ 5.7, & \xi \ge 1; \end{cases} \tag{3}$$

$$\Phi_2 = \begin{cases} 0.7\,(1 - 9\xi)^{-1/2}, & \xi < 0, \\ 1 + 4.7\xi, & 0 < \xi < 1, \\ 5.44, & \xi \ge 1; \end{cases} \tag{4}$$

$$\Phi_3 = \begin{cases} \Phi_2, & \xi < -0.5, \\ 0.93\,(1 - 9\xi)^{-1/2}, & -0.5 < \xi < 0, \\ \Phi_1, & 0 < \xi; \end{cases} \tag{5}$$

$$c_1 = 3.375\kappa, \quad \kappa = 0.4, \quad c_2 = 2.69\kappa, \quad \xi = \frac{z - z_*}{L}, \quad L = -\frac{\hat{u}_*^3\, T}{\kappa g \overline{T'w'}},$$

where $\overline{T'w'}$ is the vertical turbulent heat flux in the near-ground atmospheric layer. Another form of universal functions can be found in [12].

Measuring the mean wind velocity and temperature and moisture of the air at the two levels z_1 and z_2 in the near-ground atmospheric layer, choosing the functions $\Phi_{0i}(\xi) = 1 - \beta\xi$, $\beta \sim 0.7$, $i = 1, 2, 3$ as a zero approximation, and then solving the inverse problem (1)–(5) by successive approximations, we can find the entire set of characteristics of the turbulence in the near-ground layer of the atmosphere: z_*, L, $\hat{u}_*$, $\hat{T}_*$, $\hat{q}_*$, $T(z_*)$, and $q(z_*)$.

Heat-Moisture Transfer in the Viscous-Buffer Layer of the Near-Ground Air. By virtue of the small thickness and the immobility, on the average, of the viscous-buffer layer of the near-ground air its heat-moisture state can be written in a diffusion approximation that, however, accounts for the attenuating turbulent transfer:

$$\partial_t X_1 = \partial_z (K(z)\, \partial_z X_1) + K_{13} e^1 \langle E \partial_z X_1, \partial_z X_1 \rangle \tag{6}$$

($z \in [z_{**}, z_*]$, $0 \le t_0 \le t \le \tau$, τ is the forecast interval).

In (6), $X_1 = \mathrm{col}\,(T_{\text{med}}, \rho_1)$, $T_{\text{med}} = T_{\text{med}}(z, t)$, $\rho_1 = \rho_1(z, t)$ is a vector whose coordinates are the air temperature and the vapor density in the viscous-buffer layer; $K(z) = CH(z) + K_1$ is the matrix of the heat and moisture transfer coefficients; $C = \begin{pmatrix} c_{TT} & c_{Tq} \\ c_{qT} & c_{qq} \end{pmatrix}$ is the matrix of the coefficients of turbulent heat and moisture transfer attenuating in the layer [12],

$$\eta_T(z) = 0.44\hat{u}_* z\left(1 - \exp\left(\frac{z}{B_T}\right)\right), \quad B_T = \frac{\nu}{\hat{u}_* \sqrt{\Pr}} \sum_{i=1}^{5} c_i (\log \Pr)^{i-1},$$

$$c_1 = 34.96, \quad c_2 = 28.79, \quad c_3 = 33.95, \quad c_4 = 6.3, \quad c_5 = -1.186, \quad \Pr = \frac{\nu c_p \rho_{\text{med}}}{\lambda_{\text{med}}} \ (\sim 0.7).$$

Similarly,

$$\eta_q(z) = 0.44\hat{u}_* z\left(1 - \exp\left(\frac{z}{B_q}\right)\right), \quad B_q = \frac{\nu}{\hat{u}_* \sqrt{\mathrm{Sc}}} \sum_{i=1}^{5} c_i (\log \mathrm{Sc})^{i-1}, \quad \mathrm{Sc} = \frac{\nu}{D_{10}},$$

$$K_1 = (k_{1ij})_{i,j=1}^{2}, \quad k_{111} = \nu \Pr, \quad k_{112} = \frac{k_T \nu}{c_{\text{w}} \lambda_{\text{med}} (\mathrm{Sc})(\Pr)}, \quad k_{121} = \frac{\nu \rho_{\text{med}}}{T_{\text{med}} (\mathrm{Sc})}, \quad k_{122} = \frac{\nu}{\mathrm{Sc}},$$

$$K_{13} = \frac{c_p \nu^2}{\lambda_{\text{med}} (\mathrm{Sc})(\Pr)}, \quad e^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$\langle X, Y \rangle$ is the scalar product of the vectors X and Y.

We will supplement system (6) with the following boundary and initial conditions:

$$z = z_*: \quad T_{\text{med}}(z, t) = T_*(t), \quad \rho_1(z, t) = \rho_1(z_*, t) = \frac{q_*(t)\, p_*(t)}{RT_*(t)\,(1 + 0.6 q_*(t))}, \tag{7}$$

$p_*(t)$, $T_*(t)$, and $q_*(t)$ are the pressure, temperature, and specific moisture of the air at the level $z = z_*$ at the instant t;

$$z = z_{**}: \quad A_1 \partial_z X_1 = B_1 (X_2^{\varepsilon})\, \partial_z X_2^{\varepsilon} + e^1 (1 - \alpha_s) R_\delta(t) + e_1 Q(t) \equiv F_1(\partial_z X^{\varepsilon}_{2z_{**}}(t)); \tag{8}$$

$$t = t_0: \quad X_1(z, t) = X_1(z, t_0) = X_{10}(z). \tag{9}$$

In Eqs. (8) of the heat and moisture balances at the atmosphere–soil interface

$$A_1 = (a_{1ij})_{i,j=1}^{2}, \quad a_{111} = \frac{\lambda_{\text{med}} (\Pr)^2}{z_{**}}, \quad a_{112} = 0, \quad a_{121} = \frac{\nu \rho_{\text{med}}}{z_{**} T_{\text{med}} (\mathrm{Sc})}, \quad a_{122} = \frac{\chi \nu}{z_{**} (\mathrm{Sc})},$$

$X_2^{\varepsilon} = \mathrm{col}\,(T_s^{\varepsilon}, w^{\varepsilon})$ is a vector with the coordinates $T_s^{\varepsilon} = T_s(z, t, \varepsilon)$ and $w^{\varepsilon} = w(z, t, \varepsilon)$,

$$B_1(X_2^\varepsilon) = (b_{1ij}(X_2^\varepsilon))_{i,j=1}^2, \quad b_{111} = \lambda_s(w^\varepsilon, T_s^\varepsilon), \quad b_{112} = \Xi\,\chi D_s(w^\varepsilon, T_s^\varepsilon), \quad b_{121} = 0,$$

$$b_{122} = (1 - \Xi)\,\rho_w k_s(w^\varepsilon, T_s^\varepsilon), \quad e_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad 0 \le \Xi = H(w) - \beta H(w^\varepsilon - c(t)) \le 1,$$

$$\beta = \left| \frac{\partial_t w^\varepsilon}{\partial_t c} \right|$$

and for simplicity's sake $c(t) = ct$ [13, 14]. For $\Xi = 0$ phase transformations in the soil are absent and all the moisture moves in liquid form. For $\Xi = 1$ only mass transfer of vapor occurs.

According to [15–17], the heat-moisture state of the soil massif is determined as the solution of the initial-boundary-value problem

$$\varepsilon A_2(X_2^\varepsilon)\,\partial_t X_2^\varepsilon = \partial_z\,[K_2(X_2^\varepsilon, \varepsilon)\,\partial_z X_2^\varepsilon] + \partial_z\,[e^1 k_s(\langle e^1, E X_2^\varepsilon \rangle)]\,\partial_z p + \gamma \left[e_1 + \frac{\chi}{c_p} e^1 \right] \langle e^1, E\partial_t X_2^\varepsilon \rangle, \tag{10}$$

$$z = z_{**}: \quad B_1(X_2^\varepsilon)\,\partial_z X_2^\varepsilon = A_1 \partial_z X_1 - e^1 (1 - \alpha_s)\,R_\delta(t) - e_1 Q(t) \equiv F_2(X_{1z_{**}}, t), \tag{11}$$

$$z = -h: \quad X_2(t, z, \varepsilon) = X_{2,-h}^\varepsilon(t), \tag{12}$$

$$t = t_0: \quad X_2(t, z, \varepsilon) = X_{20}^\varepsilon(z), \tag{13}$$

supplemented with an equation of state that relates the moisture content to the pressure and temperature for a prescribed geometric structure of the pore space of the soil massif:

$$\langle e^1, E X_2^\varepsilon \rangle = f(T_s^\varepsilon, p^\varepsilon, \varepsilon). \tag{14}$$

In (10)–(13)

$$A_2(X_2^\varepsilon) = (a_{2ij}(X_2^\varepsilon))_{i,j=1}^2, \quad a_{211} = \rho_s c_s, \quad a_{212} = 0, \quad a_{221} = 0, \quad a_{222} = \rho_w = 1,$$

$$K_2(X_2^\varepsilon) = (k_{2ij}(X_2^\varepsilon))_{i,j=1}^2, \quad k_{211} = \lambda_s(X_2^\varepsilon), \quad k_{212} = 0, \quad k_{221} = \rho_w D_s(X_2^\varepsilon)\,\delta_s(X_2^\varepsilon, \Xi),$$

$$k_{222} = \rho_s D_s(X_2^\varepsilon), \quad \gamma = \varepsilon \rho_1 \Xi$$

Now we will impose on the solutions of the initial boundary-value problem (6)–(14) continuity conditions for the temperature and moisture distributions and conditions of finiteness of the discontinuities of their gradients at the interface of the media by virtue of the dissimilar physical properties of air and soil:

$$T_{\rm med}(t, z_{**} + 0) = T_s^\varepsilon(t, z_{**} - 0), \tag{15}$$

$$\begin{aligned} [\partial_z T(t, z_{**})] &= \partial_z T_{\rm med}(t, z_{**} + 0) - \partial_z T_s^\varepsilon(t, z_{**} - 0) \ne \pm\infty, \\ [\partial_z q(t, z_{**})] &= \partial_z q(t, z_{**} + 0) - \partial_z w^\varepsilon(t, z_{**} - 0) \ne \pm\infty. \end{aligned} \tag{16}$$

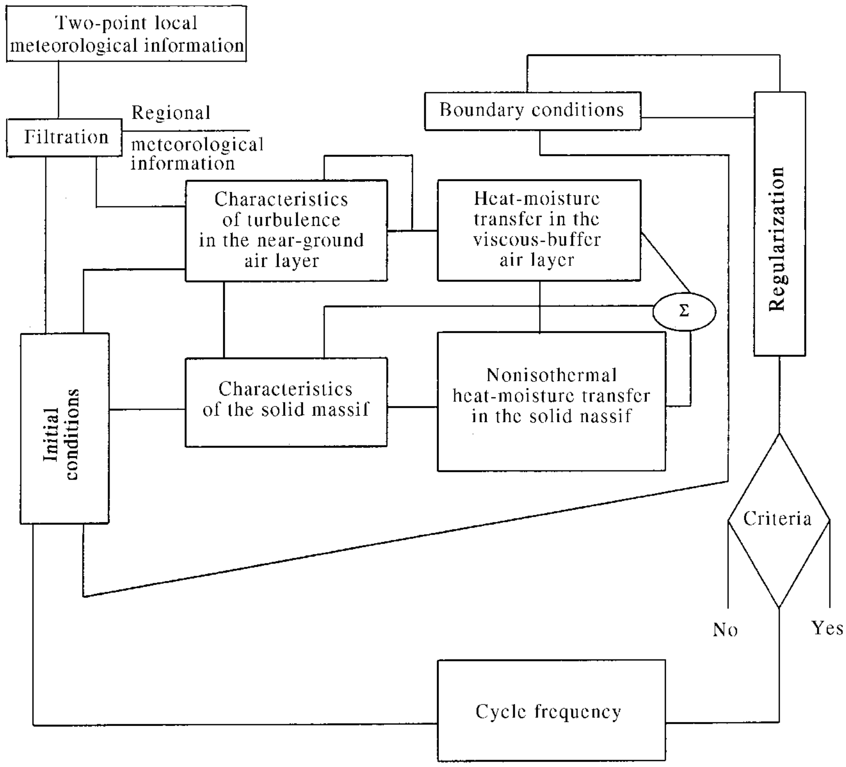

Fig. 1. Block diagram of the solution procedure.

Let z_{*wg} = min $\{z \in [-h, z_{**}]: z = z_{wg}, \Xi \neq 0\}$ be the lower bound of the set of points z_{wg} at which the water–vapor phase transition occurs and z_{*gg} be the upper bound of the same set. By $t^{**} \in [t_0, \tau]$ we denote the moment at which one of the following conditions is fulfilled on the hyperplane $z = z_{**}$ for the solution of system (6)–(14):

$$\text{a)}\ T_s^{\varepsilon}(z_{**}, t^{**}) \le 0\,,\quad p_1(z_{**}, t^{**}) \ge p_{sat}\,, \tag{17}$$

$$\text{b)}\ T_s^{\varepsilon} > 0\,,\quad z_{**} = z_{*wg}\,,\quad t = t^{**}\,, \tag{18}$$

$$\text{c)}\ T_s^{\varepsilon} > 0\,,\quad z_{**} = z_{*gg}\,,\quad t = t^{**}\,, \tag{19}$$

where $p_1(z_{**}, t^{**})$ is the partial vapor pressure at the soil–atmosphere interface $z = z_{**}$ and $p_{sat} = 133.32 \cdot 10^{\frac{8.127 T_s(z_{**}, t^{**}) + 156}{T_s(z_{**}, t^{**}) + 236}}$ is the Filnei formula of one-to-one correspondence between the partial pressure of the saturated vapor and its temperature [18].

In the case of condition (17), from the Clausius–Clapeyron equation [5] it follows that on the soil surface there is water in the solid phase, i.e., frost. In the case of condition (18) the soil layer $[-h, z_{**}]$ will contain moisture in liquid form, while in the case of condition (19) moisture in liquid form is absent at the soil–atmosphere interface.

Method of Solution of the Problem. We describe in brief a method for solving problem (6)–(14) with free (unknown, determined by solution) conditions (12)–(13). A block diagram of the procedure is shown in Fig. 1.

We represent the coefficients of transfer from the "lower system" (10) in the form $\varphi(X) = \varphi_0 + \varphi_1(X)$; these coefficients depend nonlinearly on the running heat-moisture state of the soil massif. Isolating the linear components in the right-hand sides of Eqs. (6) and (10), we will construct the matrix Green functions for the "upper" and "lower" initial-boundary-value problems formed by these components and conditions (7)–(9) and (12)–(13):

$$G^{(i)}(z, \xi, t) = \operatorname{diag}\left\{ \sum_{n=1}^{\infty} \varphi_n^{(i)}(z)\, \varphi_n^{(i)}(\xi) \exp(\lambda_r^{(i)} \mu_n^{(i)} t) \right\}_{r=1}^{2},$$

$$\operatorname{mes} \Omega^{(i)} = \begin{cases} z_* - z_{**}, & i = 1, \\ z_{**} + h, & i = 2, \end{cases}$$

where $\varphi_n^{(i)}(z) = (\operatorname{mes} \Omega^{(i)})^{-1/2} \sqrt{2} \sin(\delta_{i2}\frac{\pi}{2} - \mu_n^{(i)})z$, $z \in \operatorname{mes} \Omega^{(i)}$, and $\mu_n^{(i)}$ are the roots of the equation

$$\mu^{(i)} \tan(\mu^{(i)}) \operatorname{mes} \Omega^{(i)} = 1; \quad \lambda_r^{(i)} = \begin{cases} \nu\,(\mathrm{Pr}), & i = 1, \\ \dfrac{\nu}{\mathrm{Sc}}, & i = 2, \end{cases} \quad r = 1;$$

$$\lambda_r^{(i)} = \begin{cases} \lambda_{s0}, & i = 1, \\ D_{s0}, & i = 2, \end{cases} \quad r = 2$$

are the constant components of the heat and moisture transfer in the air and the soil and δ_{i2} is the Kronecker symbol.

Using the constructed matrix Green functions, we pass to the system of nonlinear integrodifferential equations

$$X^{\varepsilon} = V \mathfrak{I}\, \Psi(L_{(1)}(X^{\varepsilon})) + V_{z_*,-h} X^{\varepsilon}_{z_*,-h} + V_{z_{**},z_{**}} F(L_{(2)}(X^{\varepsilon}_{z_{**}})) + \mathfrak{I}_0 X^{\varepsilon}_0, \tag{20}$$

which is equivalent to the original general initial-boundary-value problem.

In expression (20)

$$X^{\varepsilon} = \operatorname{col}(X_1, X_2^{\varepsilon}), \quad V \mathfrak{I} \circ = \int_0^t \begin{pmatrix} \mathfrak{I}_{11} & 0 \\ 0 & \mathfrak{I}_{22} \end{pmatrix} \circ\, d\tau;$$

$$\mathfrak{I}_{ii} \circ = \int_{\operatorname{mes}\Omega^{(i)}} T^{(i)} G^{(i)}(z, \xi, t - \tau)\, T^{(i)-1} \circ\, d\xi;$$

$T^{(i)}$ are real orthogonal matrices that bring the matrices K_1 ($i = 1$) and $A_2^{-1}K_{20}$ ($i = 2$) to the Jordan form $J_1(K_1)$ and $J_2(A_2^{-1}K_{20})$, respectively, and Ψ is the superposition operator, $\Psi L_1 X^{\varepsilon} = \operatorname{col}(\partial_z(CH\partial_z X_1) + K_{13}e^1 \langle E\partial_z X_1, \partial_z X_1\rangle)$, $\operatorname{col}(A_2^{-1}[\partial_z(k_{21}(X_2^{\xi})\partial_z X_2^{\xi}) + j_{212}e^1\langle e_1, \partial_z X_2^{\xi}\rangle + e^1 k_s \partial_z p] + \gamma[e_1 + \frac{\chi}{c_p} e^1]\langle e^1, E\partial_t X_2^{\xi}\rangle))$, where j_{212} is the upper right-hand element of the matrix $J_2(A_2^{-1}K_{20})$;

$$V_{z_*,-h}\circ=\int_{t_0}^{t}\operatorname{diag}\,(J_1\,(K_1)\,G^{(1)}\,(z,z_*,t-\tau)\,,\;\;J_2\,(A_2^{-1}K_{20})\,G^{(2)}\,(z,-h,t-\tau))\circ d\tau\;;$$

$$V_{z_{**},z_{**}}\circ=\int_{t_0}^{t}\operatorname{diag}\,(J_1\,(K_1)\,G^{(1)}\,(z,z_{**},t-\tau)\,,\;\;J_2\,(A_2^{-1}K_{20})\,G^{(2)}\,(z,z_{**},t-\tau))\circ d\tau\;;$$

$X^{\varepsilon}_{z_*,-h}(t)$ = col $(X_1(z_*, t), X^{\varepsilon}_2(-h, t))$ is the vector of the temperature and moisture values at the external boundaries of the problem $z = z_*$ and $z = -h$;

$$F\,(L_{(2)}\,X^{\varepsilon}_{2z_{**}})\,(t)=A_1^{-1}\,[B_1\,(X_2^{\varepsilon}\,(z_{**},t))\,\partial_zX_2^{\varepsilon}\,(z_{**},t)+e^1\,(1-\alpha_s)\,R_{\delta}\,(t)+e_1Q\,(t)]\;;$$

$X^{\varepsilon}_{z_{**}}(t)$ is the vector of the temperature and moisture values at the internal boundary of the problem, i.e., on the hyperplane of the soil–atmosphere interface, $\Im_0\,\circ\,=\,V\,\Im\,\Delta_{t,t_0}\circ\,=\,\Im V\Delta_{t,t_0}\circ$, $\Delta_{t,t_0}=\delta(t-t_0)I$ is the matrix δ-function, and $X_0(z) = X(z, t_0)$ is the initial distribution of the temperatures and moisture in the above-ground and underground layers of the system of soil–atmosphere.

We will differentiate system (20) with respect to z at the point $z = z_{**}$ and obtain four more equations for the temperature and moisture gradients of the air and soil at the soil–atmosphere interface:

$$\partial_zX^{\varepsilon}=V\,\Im^{\wedge}\Psi\,(L_{(1)}X^{\varepsilon})+V^{\wedge}_{z_*,-h}\,X^{\varepsilon}_{z_*,-h}+V^{\wedge}_{z_{**},z_{**}}\,F\,(L_{(2)}X^{\varepsilon}_{z_{**}})+\Im_0^{\wedge}X_0^{\varepsilon}\,. \tag{21}$$

In (21) the integral operators $\Im^{\wedge}$, $V^{\wedge}_{\cdot,\cdot}$, and $\Im_0^{\wedge}$ have the same structure as the integral operators $\Im$, $V_{\cdot,\cdot}$, and $\Im_0$ and are found from the latter by applying the operator ∂_z to their kernels and then calculating the obtained matrix functions at the point $z = z_{**}$.

We introduce into our considerations the two orthoprojectors $P=\begin{pmatrix}I_2 & 0\\ 0 & 0\end{pmatrix}$ and $Q = I_4 - P$ onto the two-dimensional subspaces of the four-dimensional phase space of problem (6)–(14) that correspond to the phase spaces of the "upper" and "lower" subsystems (6)–(9) and (10)–(14), respectively. Implementing the conditions of continuous conjugation of the solutions of the "upper" and "lower" problems, we will apply the operators P and Q to the right-hand sides of system (20) and then equate the obtained two-dimensional vector functions at the point $z = z_{**}$:

$$PX^{\varepsilon}\,(z_{**},t)=QX^{\varepsilon}\,(z_{**},t)\,. \tag{22}$$

The system of equations (20)–(22) allows complete determination of the heat-moisture state of the near-ground layer of air and the near-surface layer of soil at the heights $[-h, z_*]$ in the time interval $[t_0, \tau]$.

A numerical solution of system (20)–(22) is sought in the form of a series in the eigenvectors of Green's kernels $G^{(i)}(z, \xi, t)$ of the operators generated by this system, while a correspondence is set up between the system (20)–(22) itself and the infinite-dimensional (calculating) evolution system for the coefficients of the expansion of the solution into the indicated series.

To the evolution infinite-dimensional systems for the expansion coefficients the operations of discretization, linearization, and reduction to a finite-dimensional system are applied; the latter, in turn, becomes regularized and its solution, being a finite-dimensional approximation to the solution of the initial system (20)–(22), is found by a modified Newton method with control [8].

A special feature of the entire algorithm for forecasting the heat-moisture state of the soil–atmosphere interface is continuous functioning in a cyclic mode up to the moment of the onset of the forecasted event or

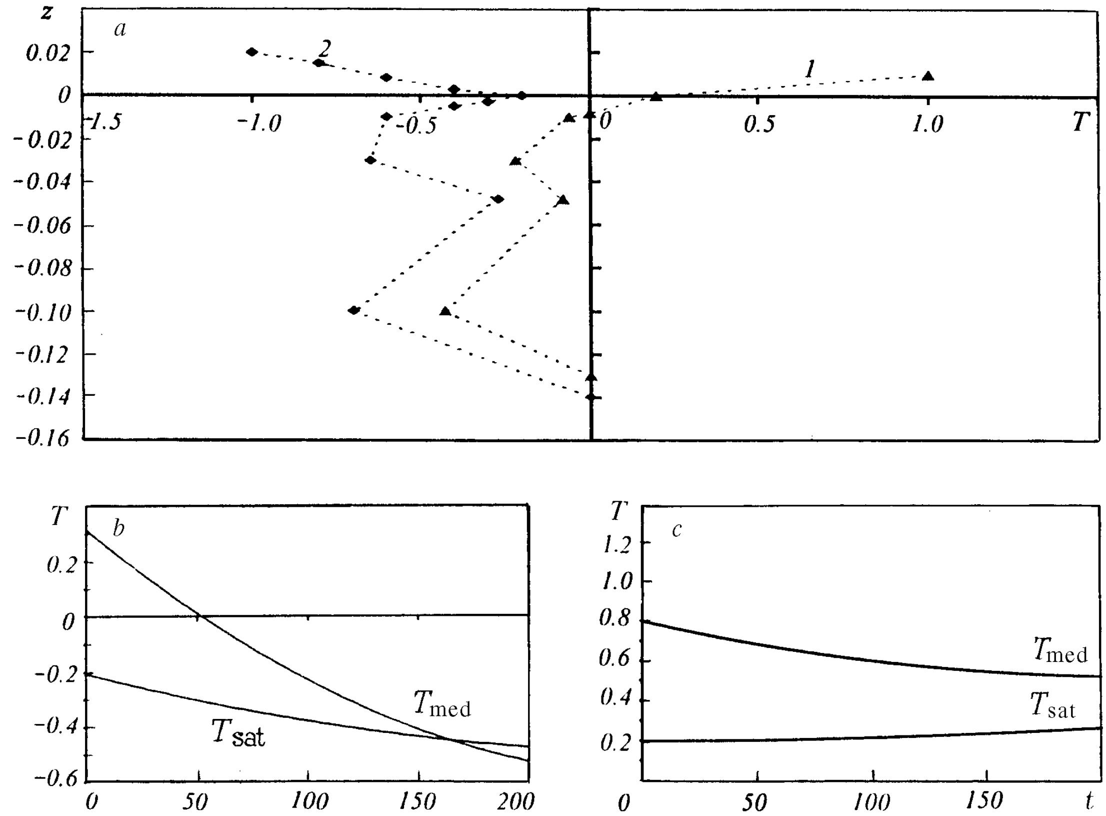

Fig. 2. Dynamics of the interaction of a runway and the atmosphere: a) temperature distribution (^{o}C) over the height (m) of the system of runway–viscous-buffer atmospheric layer for 09.01.1994 [1) at 12 h 30 min; 2) at 15 h 05 min]; b, c) time dependence of the air temperature T_{med} and the saturation temperature T_{sat} (^{o}C) in the viscous-buffer atmospheric layer above the runway for the cases of ice formation (b) and its absence (c).

up to a certain instant t_N, after which there is no need to forecast. The heat-moisture state of the viscous-buffer layer of air and the near-surface layer of soil is calculated continuously on the intervals $[t_0 + n\omega, \tau_n]$, $\tau_n = t_0 + n\omega + \tau$, $n = 0, 1, ...$, ω is a certain period, prescribed in accordance with the demand of a forecast consumer or of the developing weather conditions, for the arrival of new local meteorological information on the state of the near-ground layer and information on the state of the system at the end of the preceding step. It is obvious that with such organization of the forecast the probability of the prediction tends to unity as the moment of onset of the event is approached.

An Application of the Results. The above approach and the theoretical considerations are implemented, in particular, in a computer system, created by the Agrophysical Institute and intended for forecasting ice formation on runways of airfields. The system has been set up and has been operating successfully for more than two years at Pulkovo airport, St. Petersburg. According to the requirements of safety and reliable service of runways, in developing the system it has been assumed that: 15 min $\le \tau \le$ 3 h, the probability of reliable prediction must be no lower than 0.85, the period of cyclic functioning of the system is chosen to be the maximum possible, ω = 15 min, meteorological measurements on a meteorological mast in the near-ground layer of air near the runway are carried out on the levels z_1 = 0.5 m and z_2 = 4 m above the runway surface.

Figure 2a shows a vertical temperature profile in the runway body and in an air layer adjacent to it, while Fig. 2b and c provides the time dependence of the mean-integral air temperatures and saturation in a viscous-buffer layer at the runway surface in operation of the system for cases of occurrence and absence of ice formation.

NOTATION

z, vertical coordinate; t, time; $u(z)$, $T(z)$, and $q(z)$, vertical distributions of the mean velocity, temperature, and specific humidity of the air in the near-ground atmospheric layer; L, $\hat{u}_*$, $\hat{T}_*$, and $\hat{q}_*$, characteristic scales of length, velocity, temperature, and moisture in the turbulent sublayer of the near-ground atmospheric layer; $(\&')$, centered pulsation of the quantity (&); $((\$)'(\&)')$, correlation of pulsations of the quantities ($) and (&); Pr, molecular Prandtl number; v, ρ_{med}, λ_{med}, and T_{med}, kinematic viscosity, density, thermal conductivity, and temperature of the air in the viscous-buffer layer; ρ_1 and p_1, density and partial pressure of the vapor in the viscous-buffer layer; Sc, Schmidt number; D_{10}, coefficient of molecular diffusion in moist air; c_{p_1}, c_p, and c_w, heat capacities of the vapor and the air at constant pressure and heat capacity of the water; $\frac{K_T}{c_w}$, specific external heat of vaporization; ∂_t and ∂_z, operators of time and space differentiation; R, gas constant of the air; $(\&)_*$, value of the quantity (&) at the level $z = z_*$; $(\&)_{**}$, value of the quantity (&) at the level $z = z_{**}$; χ, specific heat of vaporization; α_s, albedo of the soil surface; R_δ, radiative balance on the soil surface; Q, intensity of the atmospheric precipitation; T_s^ε and w^ε, temperature of the soil and moisture content in it; ρ_s, λ_s, D_s, and k_s, averaged density of the soil and its coefficients of thermal conductivity, moisture conductivity, and filtration; Ξ, criterion of the phase transitions in the soil massif; $H(^*)$, Heaviside function; c, evaporation rate in the soil massif; c_s and p^ε, heat capacity and generalized water potential of the soil; δ_s, coefficient of thermogradient transfer of moisture in the soil; col $(\&_1, \&_2, ..., \&_n)$, vector with the coordinates $\&_i$, $i = 1, 2, ..., n$; mes Ω, measure of the set Ω; $J(\square)$, Jordan form of the matrix $\square$; $\square^{-1}$, inverse matrix; I, unit matrix; V°, transform of the operator V $((^{\circ})$, inverse transform of the operator V). Subscripts: med, medium; sat, saturation; w, water; g, gas; s, soil.

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